Problem 1.7

Show that the multiplication of a vector $z = Ae^{i\omega t}$ by i rotates it by 90°.

Solution

Represent i in polar form by using Euler's formula.

$$i = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = \exp\left(i\frac{\pi}{2}\right)$$

Now multiply i and z together.

$$\begin{aligned} iz &= \exp\left(i\frac{\pi}{2}\right)(Ae^{i\omega t}) \\ &= A\exp\left(i\frac{\pi}{2} + i\omega t\right) \\ &= A\exp\left[i\left(\frac{\pi}{2} + \omega t\right)\right] \end{aligned}$$

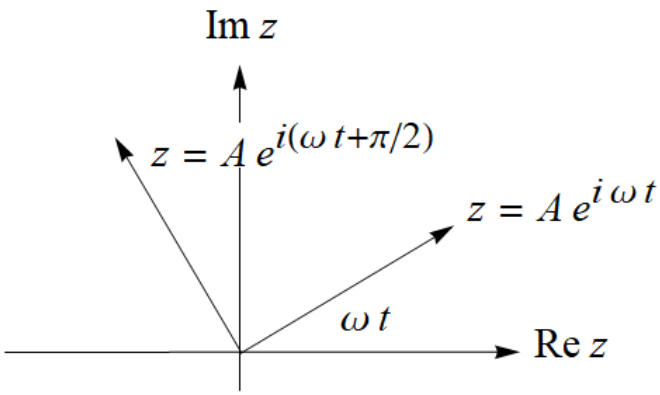

Figure 1: This figure shows the effect of multiplying a vector $z = Ae^{i\omega t}$ by i in the complex plane. It results in a counterclockwise rotation by 90°.